HYDRAULIC RESISTANCE OF CHANNELS WITH PERMEABLE WALLS

I. S. Kochenov and O. Yu. Novosel'skii

UDC 532.54:532.501.312

We present the results from an experimental study of hydraulic resistance in turbulent flow for an incompressible liquid in channels (collectors) of constant cross section, with a runoff and influx through a permeable (perforated) wall.

Calculations of the change in pressure through the length of a channel (collector) during the runoff or influx through a wall must be based on the use (in some form) of the equations of liquid motion with a variable flow rate along the path [1, 2]:

$$\frac{dp}{\rho} + \beta w dw + w^2 d\beta + \beta w^2 \frac{dG}{G} + \zeta \frac{w^2}{2} \frac{dx}{D_{eq}} = 0. \tag{1}$$

Equation (1) differs from the conventional Bernoulli equation primarily in the presence of the term $\beta w^2 dG/G$ by means of which we take into consideration the dynamic or inertial effect of mass separation or addition. This inertial term is frequently the most important in the equation and failure to make provision for this term leads to an error which, as will be demonstrated below, may amount to 100% and more of the calculated pressure difference.

For specific calculations with (1) we have to know the relationship applicable to the frictional resistance factor ζ in that equation (this is some dimensionless equivalent of the shearing stress τ at the wall):

$$\zeta = \frac{8\tau}{\rho w^2} \tag{2}$$

and for the momentum-flux factor

$$\beta = \frac{1}{F} \int_F \left(\frac{u}{w}\right)^2 dF. \tag{3}$$

The coefficients ζ and β are [3, 4] functions of the local Reynolds number $Re(x) = w(x)D_{eq}/\nu$ and of the runoff coefficient $K_\perp(x) = v_w(x)/w(x)$.* However, experimental data on these coefficients are presently few in number and contradictory [5, 6]. This makes it necessary to apply inadequately validated assumptions in the calculations of the change in pressure along the collectors as regards the coefficient of hydraulic resistance [7, 8].

There are even fewer data on the momentum-flux coefficient β.

The experimental determination of the coefficients $\zeta(Re, K_\perp)$ and $\beta(Re, K_\perp)$ is an extremely difficult problem, since it is impossible to find both of these coefficients from the data of a one-dimensional experiment without measuring the shearing stress at the wall (with the use Eq. (1)). For this we require an additional measurement of τ and of the velocity profile for u(r, x).

It is therefore natural that an attempt be made to reduce (1) to an equation with a single unknown which can be found from a one-dimensional experiment without measurement of τ.

*Here and below in the case of runoff the quantities v_w and, consequently, $K_\perp$ are positive, while in the case of influx they are negative.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 16, No. 3, pp. 405-412, March, 1969. Original article submitted May 22, 1968.

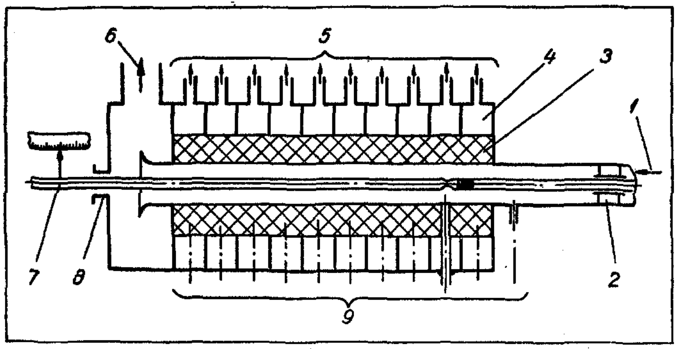

Fig. 1. Diagram of the working section: 1) water inlet; 2) support for pulse tube; 3) porous Plexiglas tube; 4) chamber; 5) rotameter drains; 6) water outlet; 7) movable pulse tube; 8) packing gland; 9) pressure probes at pulse orifices in the wall (to a battery manometer).

It was proposed in [9] that a study be undertaken with regard to the use, in subsequent calculations, of the local ratio of the Euler numbers – the true (experimental) and theoretical – as such a coefficient:

$$E = \frac{\mathrm{Eu}}{\mathrm{Eu_{th}}}, \tag{4}$$

where

$$\mathrm{Eu_{th}} = 16\mathrm{K}_\perp - \zeta_0.$$

This method can be applied successfully in calculating flows with high runoff coefficient; however, it is not suitable near values of $\mathrm{K}_\perp = \zeta_0/16$.

We therefore attempted to develop a more universal method that is also based on application of the equation of motion (1), but devoid of this drawback.

In dimensionless form Eq. (1) can be written as

$$-\mathrm{Eu} = \frac{dP}{dX} = 16\beta\mathrm{K}_\perp\left(1 + 0.5\,\frac{d\ln\beta}{d\ln\mathrm{Re}}\right) - \zeta. \tag{5}$$

Bearing in mind that β is close to unity and that the known portion must be excluded from the object of our study, let us rewrite (5) in the form

$$\frac{dP}{dX} - 16\mathrm{K}_\perp = 16\mathrm{K}_\perp(\beta - 1) + 8\mathrm{K}_\perp\mathrm{Re}\frac{d\beta}{d\mathrm{Re}} - \zeta. \tag{6}$$

Having denoted the right-hand member of (6) as ξ, we find the equation of motion in a simple form that is convenient for calculation:

$$\frac{dP}{dX} = 16\mathrm{K}_\perp - \xi. \tag{7}$$

The variable ξ will be known as the effective coefficient of hydraulic resistance.*

For flow with runoff (distribution collectors) we have $\beta \approx 1.0$, and the hydraulic-resistance factor ξ is approximately equal to the coefficient of frictional resistance ζ. For flows with influx (receiving collectors) the coefficient ζ is small in most cases [6]. In this case ξ may depend strongly on the coefficient ξ for the momentum flux; however, in terms of absolute magnitude ξ is small.

Analysis of the right-hand member of (6) easily demonstrates that ξ (as well as ζ and β) is a function of the local criteria $\mathrm{K}_\perp(x)$ and $\mathrm{Re}(x)$ (or $\mathrm{K}_\perp(x)$ and $\mathrm{Re}_\perp(x) = v_w D_e/\nu$).

To study ξ we develop an experimental installation whose working section is shown in Fig. 1; the working fluid is water. The basic element of the working section is a thick-walled Plexiglas tube with an inside

*Or for brevity, simply as the coefficient of hydraulic resistance.

TABLE 1. Parameters of Porous Tubes

No. of working section	1	2	3	4	5
Porosity ε_f	0,008	0,024	0,080	0,160	0,030
No. of orifices per row	4	4	8	16	2 and 3
No. of orifice rows	10	30	50	50	50
Mean relative pitch t/D_{eq}	1,8	0,578	0,346	0,346	0,346
Length of working section, mm	225	241	241	241	241
Value of central angle φ, within the limits of which orifices were drilled.	360° (drill holes about entire perimeter)				90°

diameter of 20 mm, whose porosity is governed by the number of orifices (2 mm in diameter), drilled into the wall.

The parameters of the porous tubes on which the tests were carried out are given in Table 1.

The basic requirements imposed on the experiment to find the resistance factor involve the exact establishment and maintenance of a specified runoff (influx) law over the length of the channel $dG/dx = f_1(x)$ and the exact determination of the derivative $dp/dx = f_2(x)$. To satisfy the first of these requirements, we have made the working section in 10 segments. Each segment is connected by means of a regulation valve to its own rotameter to set and measure the required flow rate for the runoff (influx) through the given segment.

To raise the reliability of the procedure by which the static pressure is measured along the length of the porous tube, the measurement system was duplicated. The pressure was sampled at the pulse orifices in the wall (one orifice per segment) and at the movable pulse tube, the latter provided with a positioning device (Fig. 1). With the pulse tube we can measure the static pressure for virtually any pitch. The readings of both systems were fed to a battery-operated water differential manometer.

In studying flow with runoff, the basic readings were those from the movable pulse tube, while in studying flow with influx, the basic readings were those taken from the pulse orifices at the walls of the various segments (the readings of the pulse tube, particularly when $K_\perp < -0.08$, were distorted by the velocity head of the jet of incoming fluids).

Because of the use of the movable pulse tube, the channel of the working section was made annular, with a diameter ratio of 0.305 and an equivalent diameter of 13.9 mm.

The friction factor ζ and the momentum-flux coefficient β for flows with a relatively smooth change in the liquid flow rate over the length are virtually independent of the form of the function G(x) [3, 4, 6]. It follows from (6) and (7) that the hydraulic-resistance factor in this case must also be virtually independent of the law governing the change in flow rate over the length of the collector. The basic experiments in this effort were therefore performed with the influx and runoff uniform over the length, while the validity of applying the derived relationships to any other (but sufficiently smooth) distribution function G(x) is demonstrated by special experiments.

Processing of the measurement results was accomplished on an electronic digital computer. The experimental values of p(x) were approximated by a polynomial of third degree whose coefficients were determined by the method of least squares. The degree of the polynomials was selected on the basis of data from a preliminary theoretical analysis [1], as well as an analysis of the primary experimental results. The derivatives dP/dX were calculated at the points at which the static pressure was measured.

The hydraulic-resistance factor ξ was determined from (7), and this led to the question as to which runoff (influx) coefficient it is best to use, i.e., the one calculated in terms of the "true" velocity $v_{true} = \Delta G/\rho\pi D\Delta x\varepsilon_f$ at the orifices of the porous tube, or the one calculated in terms of the conditional velocity $v_{con} = \Delta G/\rho\pi D\Delta x$, referred to the entire side surface. We will refer to the runoff (influx) coefficient calculated by the first method (from v_{true}) as the "true" coefficient and we will denote it as $K_\perp$true, while the coefficient determined by the second method will be known as the "conditional" coefficient, denoted as $K_\perp$.

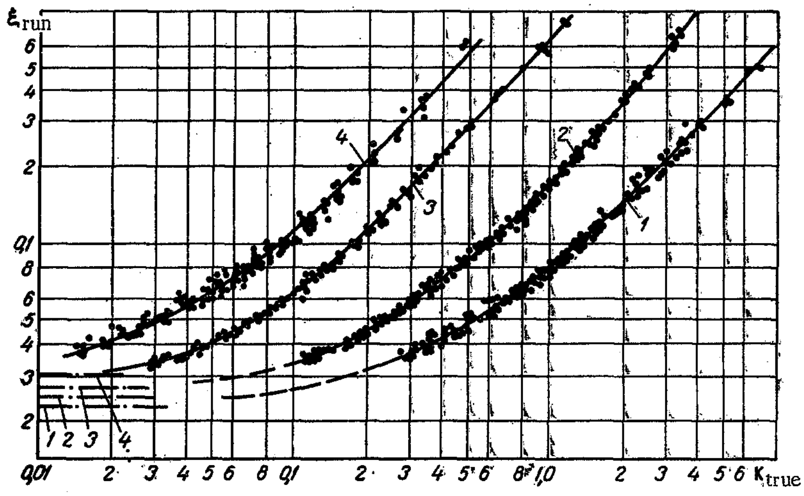

Fig. 2. Effective coefficient of hydraulic resistance for flow with runoff as a function of the "true" runoff coefficients: 1) $\varepsilon_f = 0.008$; 2) 0.024; 3) 0.08; 4) 0.16 (the dash-dot lines show the corresponding values of ζ_0).

TABLE 2. Values of the Coefficient and Exponents in (8)

Porosity ε_f	0,008	0,024	0,080	0,160
a	0,0532	0,143	0,609	1,601
b	1,15	1,24	1,27	1,28
ζ_0	0,0235	0,0253	0,273	0,0301

Results from the processing of the experimental data with respect to flows involving runoff are shown in Fig. 2 as functions of $\xi_{run} = \xi(K_{\perp true})$ (from the Reynolds number, when $Re > 10^4$ the resistance factor ξ is self-similar). The experimental points provide a good approximation of the exponential relation

$$\xi_{run} = \zeta_0 + aK^b_{\perp true}. \tag{8}$$

The coefficients of (8), found by the method of least squares, are shown in Table 2.

The calculated values of ζ_0 agree with the self-similar values of the friction factor for the porous tube, these having been derived experimentally with constant mass flow through the tube (when $K_\perp = 0$).

In connection with the fact that the walls of the porous tube exhibit substantial roughness because of the orifices, the self-similarity of ζ_0 – on the basis of the Reynolds number – was reached as soon as $Re = (3-4) \cdot 10^4$.

It proved to be preferable (more universal) to process the experimental data in the variables $\xi_{run} = \xi(K_\perp)$, i.e., on the basis of the conditional runoff coefficient ($K_\perp = K_{\perp true}\varepsilon_f$). Here the experimental points for the three variants of the porous tube, with the exception of the first, are generalized by the single relationship

$$\xi_{run} = \zeta_0 + 15.6 \cdot K_\perp^{1.27}. \tag{9}$$

Formula (9) in the range of parameters $10^4 \leq Re \leq 1.1 \cdot 10^5$; $0.0022 \leq K_\perp \leq 0.1$; $t/D_{eq} \leq 0.578$, and $d/D_{eq} \leq 0.145$ describes the experimental points with a standard deviation of $\sim 6\%$. The hydraulic-resistance factor in flow with runoff in the cited range of parameters is virtually independent of the porosity and relative spacing between the orifice rows. It is only ζ_0 that depends weakly on these quantities.

Under actual conditions, we most frequently encounter a discrete change in the flow rate of the liquid over the length, and the fundamental difference between the real flow and that of a mathematical model (with a constant runoff over the length and the perimeter) is determined by the magnitude of the relative pitch t/D_{eq}. Here the effect of t/D_{eq}, provided it is sufficiently large, becomes noticeable in the intial experimental results. Thus, when $t/D_{eq} = 1.8$ the piezometric lines are not smooth. Between the orifice rows, over the length t of the pitch, the flow can undergo changes designed to "restore" the characteristic velocity field for a constant-mass flow, and here we either find a drop in the static pressure (with an overall rise in pressure) or a significant retardation of its increase. For a relative pitch $t/D_{eq} \leq 0.578$ this phenomenon

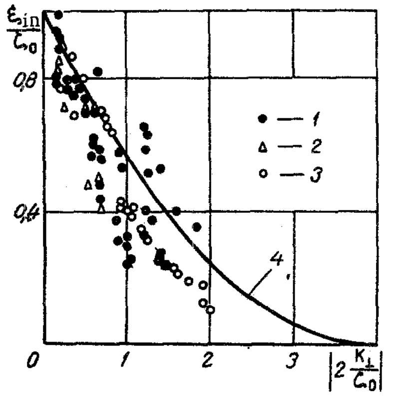

Fig. 3

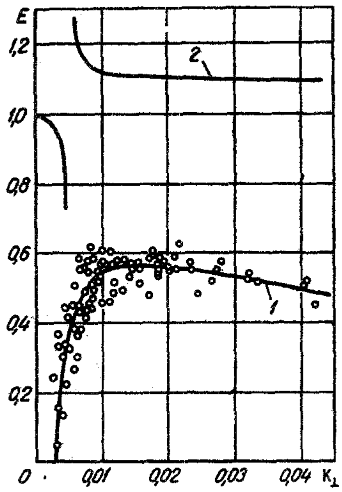

Fig. 4

Fig. 3. Comparison of experimental results with the calculation according to formula (10): 1) author's data; 2) of [6]; 3) Cf/Cf_0 (data of [12]); 4) according to (10).

Fig. 4. Relative gradients E_{run} and E^* as functions of the "conditional" runoff coefficient ($\varepsilon f = 0.16$): 1) E_{run} is the relative pressure gradient according to (4); 2) E^* is the relative pressure gradient according to (12).

is not observed. Processing of the experimental data [10] derived for values below $t/D_{eq} = 0.84$ demonstrated that the experimental points fall satisfactorily along the curve of function (9), which we derived. Hence we can draw the conclusion that ξ_{run} is virtually independent of t/D_{eq}, at least when $t/D_{eq} \leq 0.84$.

The characteristic feature of the coefficient of hydraulic resistance for flow with influx is the decline in the latter as the absolute magnitude of the influx coefficient increases, i.e., ξ_{in} is always smaller than ζ_0 (Fig. 3). The factors responsible for this behavior of ξ_{in} are covered rather fully by the authors of [6, 11].

The substantial scattering of the experimental values of ξ_{in} is explained by the fact that it is derived from (7) as the difference of two large quantities ($dP/dX < 0$ throughout), with the result that (ξ_{in}) is smaller than the other terms of (7) by 2-3 orders of magnitude.

However, we do not need high accuracy in the resistance coefficient ξ_{in} for practical calculations, since the pressure difference in flow with influx is governed primarily by inertial forces rather than by friction (see formula (7)).

Kutateladze and Leont'ev [11] examined the limit laws of friction as $Re \to \infty$ and they derived the theoretical formula for the calculation of friction on a porous plate with continuous injection. In our notation, the Kutateladze and Leont'ev formula has the form

$$\xi_{in} = \zeta_0 \left(1 + 0.5 \frac{K_\perp}{\zeta_0}\right)^2. \tag{10}$$

In Fig. 3 we compare the experimental data from [6, 12] on ζ and our data on ξ_{in} with the calculation according to (10).

Formula (10) satisfactorily describes the experimental data and can be recommended for purposes of calculation. However, for practical purposes, considering the location of the main mass of the experimental points on the curve of Fig. 3 below the theoretical curve, we can propose simpler relationships

$$\begin{aligned} \xi_{in} &= \zeta_0 + K_\perp \quad \text{when } -K_\perp \leqslant \zeta_0, \\ \xi_{in} &= 0 \quad \text{when } -K_\perp > \zeta_0. \end{aligned} \tag{11}$$

The stabilization segment for the hydraulic-resistance factor proved to be very short (both in tests with runoff and with influx) and did not extend beyond the limits of the first segment ($x/D_{eq} \approx 2$). We found no noticeable systematic stratification of the points corresponding to the initial segments of the collector; analogous results were obtained in [6] in studying flows with influx in a porous tube.

Since the collectors in numerous industrial installations are designed so that there is no runoff or influx over the entire perimeter, it would be interesting to ascertain whether or not the orifice stagger angle φ in the collector has a pronounced effect on the hydraulic resistance.

The tests performed in the No. 5 working section (see Table 1) demonstrated that the nature of the function $\xi_{run} = f(K_\perp)$ remains the same as in the case of circular runoff; however, numerically ξ_{run} is approximately 30% higher.

From this standpoint it would also be interesting to examine the experimental data of [10]. Despite the fact that the collector in that reference had only two orifice rows in diametric array (the second time there were four rows), the experimental points – as pointed out earlier – group themselves about the curve for function (9), derived for runoff over the entire channel perimeter.

In conclusion, let us dwell in somewhat greater detail on the popular method [1, 14] of calculating the pressure difference across collectors by means of the conventional Bernoulli equation, which involves the use of the coefficient of frictional resistance for a flow without runoff.

If this calculation procedure is accurate, the resulting pressure gradient

$$E^* = \frac{\frac{dP}{dX}}{8K_\perp - \zeta_0} \qquad (12)$$

(the denominator is the gradient dP/dX, calculated from the Bernoulli equation) would always be equal to unity. Figure 4 shows $E^*(K_\perp)$, calculated from experimental data for flow with runoff (from the averaged curve for $E_{run}(K_\perp)$). Near the values of $K_\perp = \zeta_0/8$, where the denominator of (12) vanishes, the error may be as great as you please. Given sufficiently large values of $K_\perp$, the error in the pressure gradient drops to ~10%, which is a result of the fact that the errors resulting from the failure to account for the inertial term in (1) and from the incorrect specification of the friction factor have been partially offset.

For the flows in receiver collectors the pressure difference derived from the application of the conventional Bernoulli equation – as follows from (1) – is smaller than the true difference by a factor of approximately two, while in the case of circular receiving chambers [1] the difference is smaller by a factor of three.

NOTATION

p	is the static pressure;
ρ	is the density;
w	is the average velocity, $w = w(x)$;
G	is the mass flow rate, $G = G(x)$;
x	is a coordinate;
D_{eq}	is the equivalent diameter;
ζ_0	is the frictional resistance factor in a constant-mass flow;
u	is the longitudinal velocity component, $u = u(r, x)$;
F	is the area of the flowthrough section of the collector;
v_w	is the radial velocity at the wall (for runoff $v_w > 0$, and for influx $V_w < 0$);
$P = p/(\rho w^2/1)$;	
$X = x/D_{eq}$;	
D	is the diameter of the porous tube (of the collector);
t	is the distance between adjacent rows of orifices in the wall of the collector;
C_f	is the resistance factor for external streamlining of the plate with injection;
$Re = wD_{eq}/\nu$	is the Reynolds number;
$K_\perp = w_w/w$	is the runoff coefficient;
$Eu = -dP/dX$	is the Euler number;
ε_f	is the channel porosity.

LITERATURE CITED

1. I. S. Kochenov and O. Yu. Novosel'skii, At. Energiya, 23, No. 2 (1967).
2. L. A. Vulis, The Thermodynamics of Gas Flows [in Russian], Gosenergoizdat, Moscow–Leningrad (1950).
3. I. S. Kochenov and V. L. Romodanov, Inzhern. Fiz. Zh., 2, No. 11 (1959).
4. I. S. Kochenov, in: The Theory of Similarity and Its Application in Heat Engineering [in Russian], No. 139, Tr. MIIT, Moscow (1961).
5. A. I. Egorov, Information Material from the No. 2 Laboratory of Water Resources of the VODGEO [in Russian], Moscow (1961).
6. Olson and Eckert, Prikl. Mekhn., No. 1 (1966).
7. G. L. Petrov, The Hydraulics of a Variable Mass [in Russian], Izd. Khar'kovskogo Universiteta (1964).
8. I. E. Idel'chik, Inzhern. Fiz. Zh., 13, No. 6 (1967).
9. I. S. Kochenov, V. V. Vasil'ev, and L. I. Baranova, in: Heat and Mass Transfer [in Russian], Vol. 1, Izd. AN BSSR, Minsk (1965).
10. G. Noseda, L'Energia Electrica, No. 1 (1959).
11. S. S. Kutateladze and A. I. Leont'ev, The Turbulent Boundary Layer of Compressible Gas [in Russian], Izd. SO AN SSSR, Novosibirsk (1962).
12. Hacker, Jet Propulsion, 26, No. 9 (1956).
13. B. V. Petunin, The Thermal Power Engineering of Nuclear Installations [in Russian], Atomizdat, Moscow (1960).
14. M. A. Styrikovich, K. Ya. Katkovskaya, and E. P. Serov, Steam Generators at Electric Power Stations [in Russian], Izd. Energiya, Moscow (1966).